Prof. Vidya Sagar
Department of physics
Raja Singh College, Siwan

Free damped oscillation

Free oscillations:— When a body capable of oscillation is displaced from its equilibrium position and released, it begins to asclllate. The frequency of oscillations depends upon certain properties of the body. Such ascillations are called 'free oscillation' and their frequency is called the 'natural frequency' of the body. Generally, the amplitude of such asciltions gradually decreases to zero owing to the presence of certain frictional or damping forces. Therefore they are also called as damped oscillations.

Damped Simple harmonic motion:— When a vibrating body vibrates in air or in any other resisting medium the amplitude of vibration does not remain constant but decreases gradually and ultimately the body comes to rest. If the velocity is small, then the retarding force due to the medium is proportional to the first power of velocity. In such a case there are two forces acting on the vibrating particle.

(i) The restoring force proportional to displacement y given by $-sy$ where s is a constant

known as stiffness constant or spring constant. The negative sign shows that the direction of the restoring force is opposite to that of displacement and,

(ii) A retarding force proportional to the velocity $\frac{dy}{dt}$ given by $-r\frac{dy}{dt}$, where r is another constant.

$\therefore$ Total force acting on the particle

$$= -r\frac{dy}{dt} - Sy$$

If m be the mass of the vibrating particle and then the total force acting on the particle $= m\frac{d^2y}{dt^2}$, where $\frac{d^2y}{dt^2}$ is the acceleration of the vibrating particle.

$$\therefore m\frac{d^2y}{dt^2} = -Sy - r\frac{dy}{dt}$$

$$\text{or, } \frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{S}{m}y = 0$$

$$\text{or, } \boxed{\frac{d^2y}{dt^2} + 2K\frac{dy}{dt} + w_0^2 y = 0}$$

Here K is called the damping coefficient and $2K$ gives the force due to the resistance of the medium per unit mass per unit velocity. This equation represents the motion of the particle under various forces mentioned above and the vibrations are damped.